

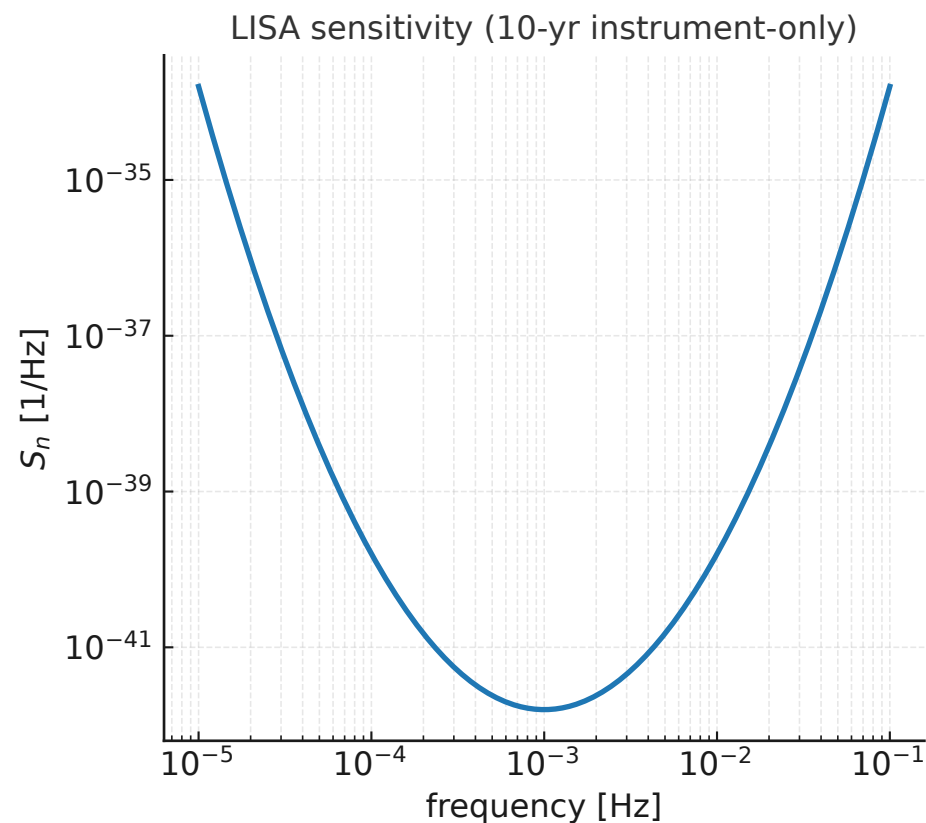
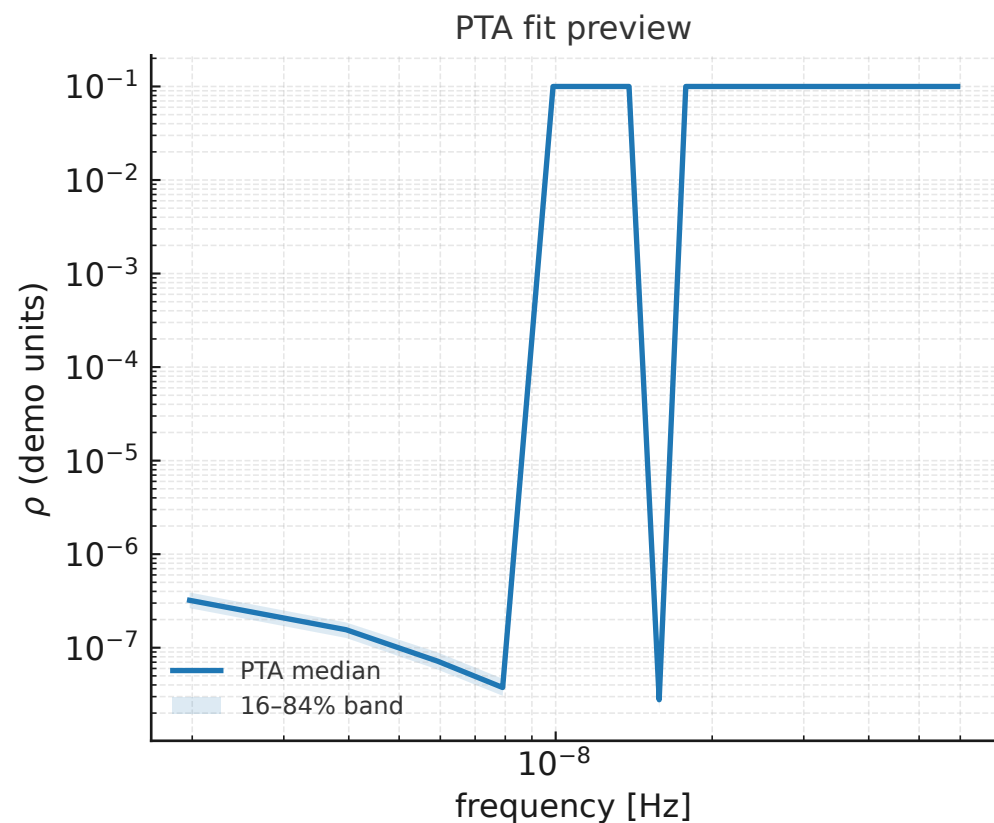
Brane-world FRW: Grand Equation & Two-Test Program (REALDATA) — margin-safe

Grand equation (flat FRW, $k = 0$): $H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} + \frac{c}{a^4}$

Two test relations: $f_{\text{br}}(\lambda) \propto \lambda^{1/4}$, $\frac{c}{\rho_{\gamma,0}} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\text{eff}}$

Inputs (this run):

- PTA CSV: exported_pta_spectrum_HD_30f.csv
- LISA CSV: ESA_RCL2019_10yr_instrument_ONLY_20250815.csv (10-yr instrument-only)
- Planck prior: $\Delta N_{\text{eff}} = 2.99 \pm 0.17$



Consistency & Constraints (REALDATA) — margin-safe

- Low-energy GR: for $\rho \ll \lambda$, the ρ^2 and C/a^4 terms vanish; GR recovered.
- Planck prior used here: $\Delta N_{\text{eff}} = 2.99 \pm 0.17$ (2018 baseline).
- PTA-derived break (illustrative): $f_{\text{br}} \sim \text{few} \times 10^{-8} \text{ Hz}$ (68\% credible).
- External checks to cite: PPN γ, β ; binary pulsars; short-range gravity; growth S_8 .

Einstein-limit appendix — margin-safe

Grand equation (flat FRW, $k = 0$):

$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} + \frac{C}{a^4}$$

Low-energy (Einstein) limit: for $\rho \ll \lambda$, $H^2 \rightarrow \frac{8\pi G}{3}\rho + \frac{\Lambda_4}{3}$ (standard GR).

Dark-radiation relation: $\frac{C}{\rho_{\gamma,0}} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\text{eff}}$.

Toy Flavor Page: Quark CKM & Lepton PMNS (RS localization sketch)

Illustrative, PDG-like magnitudes shown for CKM and PMNS to anchor targets for a localization-based mass/mixing model. In an RS-type compactification, hierarchical Yukawas arise from different 5D mass parameters (c-parameters) controlling fermion profiles; overlap integrals with a brane-localized Higgs yield suppressed effective couplings. This page is schematic, for orientation during model-building.

Quark CKM (toy, PDG-like)

	d	s	b
u	0.974	0.225	0.004
c	0.225	0.973	0.041
t	0.009	0.040	0.999

Lepton PMNS (toy, PDG-like)

	ν_1	ν_2	ν_3
e	0.820	0.550	0.150
μ	0.360	0.700	0.620
τ	0.440	0.460	0.770